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Plasma diffusion in a magnetic field

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Abstract : The effect of magnetic field on the diffusion of electrons has been studied in hydrogen, oxygen and air in the magnetic field varying 0 to 40 G within the range of pressure 0.25 to 1.6 torr. The diffusion current recorded in the direction perpendicular to the main discharge shows a sharp maximum and then gradually decreases at pressures lower than 800 mT for oxygen and air and 1250 mT for hydrogen with magnetic field. Beyond the pressure 800 mT for oxygen and air and 1250 mT for hydrogen, the diffusion current gradually decreases in the initial stage and then rapidly with the increase of the magnetic field without showing any maximum. Utilizing the diffusion current measured in the presence and the absence of magnetic field (*H*) the corresponding values of diffusion length (Λ) with or without magnetic field are determined, where pressure (*P*) is maintained as a parameter. A linearity relation between $\Lambda^2_{H} / \Lambda^2$ and H^2/P^2 has been worked out in the light of our previous paper [1]. It is shown that the aforesaid linearity relation remains valid for low values of *H/P* (<20 G/torr approx). Thus the normal diffusion loss proportional to $1/H^2$ is valid for small values of *H/P*.

Keywords : Plasma diffusion, diffusion length, magnetic field

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1. Introduction

It is fairly well established that the breakdown of a gas specially under radio frequency or microwave excitation is mainly dependent upon two principal factors, namely generation of electrons under the applied field and the loss due to diffusion and mobility and if the gas is an electron attaching one then the loss is also due to electron attachment. To study the effect of transverse or longitudinal magnetic field on the breakdown characteristics of these gases a large number of experiments over wide range of magnetic field, pressure and frequency of the applied field have been undertaken. The effect of magnetic field on the diffusion of electrons has been considered in the analysis of these results by assuming after Townsend and Gill [2] that the diffusion coefficient *D* in a direction transverse to the magnetic field is reduced according to the expression

$$D_H = \frac{D}{1+\omega_H^2\tau^2} \qquad (1)$$

where ω_H (= eH/mc) is the cyclotron frequency, τ the time of collision between successive ecounters. It can easily be deduced from eq. (1) that if Λ_H is the diffusion length of the electron in the presence of magnetic field and Λ the diffusion length in the absence of magnetic field,

then $$\Lambda_H / \Lambda = \left[1+\omega_H^2\tau^2\right]^{1/2} \qquad (2)$$

Based on eq. (2), breakdown electric field in various gases, excited by radio frequency or microwave fields in the presence of transverse magnetic field, have been calculated [3–5] and observed that some discrepancies occured between the theoretically calculated values and experimental results. It has however, been observed by Bohm *et al* [6] that some drain diffusion mechanism enabled an arc plasma to escape across the magnetic field at a speed much faster than that predicted by the normal diffusion theory. The above authors developed a theory which showed that diffusion coefficient should vary inversely as *H* and not as H^2 as shown by eq. (1). The theoretical and experimental studies of the behaviour of the positive column with stronger magnetic field and with longer tubes along with other properties have been reviewed by Bickerton and Von Engel [7], Lehnert [8] and Hoh [9]. To test the validity of eq. (2) which has been utilized in the analysis of almost all breakdown results in magnetic field it is proposed to undertake an experiment in which the diffusion length can be obtained from dc conductivity measurement of the ionised gas at different pressure and under different magnetic field. The investigation of the dependence of the diffusion coefficient on magnetic field is also considerable importance because the nature of particle loss in a hot plasma confined by a magnetic field plays a distinct role in thermonuclear reactions. With these objects in view the present work has been undertaken and results are reported in case of hydrogen, oxygen and air.

2. Experimental set-up

The experimental arrangement is shown in Figure 1. The discharge is excited in a cylindrical glass tube of length 26

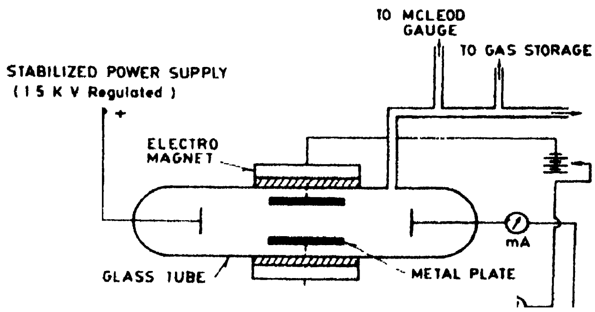

Figure 1. Experimental arrangement

cm and inner diameter 3.8 cm fitted with two circular copper electrodes each of diameter 2 cm and the distance of separation between the two electrodes being 19 cm. The regulated voltage from a stabilized power supply capable of supplying a steady voltage of 1000 V and 20 mA current excites the discharge. The rectangular brass plates (4 × 1 cm) are placed within the discharge tube with their surfaces parallel to the axis of the tube and separated by a distance of 2.5 cm. A dry battery with variable tapping (total voltage 18 V) is connected to the two brass plates through a micrometer. The tube is thoroughly cleaned and dried and placed within the pole pieces of an electromagnet so that lines of forces are perpendicular both to the length of the discharge tube as well as to the direction of the variable dc supplied to the two brass electrodes from the battery. The pole pieces have the diameter of 3.5 cm and the separation between them is 7.8 cm. The electromagnet is energized by a stabilized power supply and themagnetic field has been measured by an accurately calibrated gauss meter. The pressure of the gas was measured by a calibrated Mcleod gauge. Pure and dry air was used which was passed through phosphorous pentoxide to remove water vapour. Oxygen and hydrogen were prepared by the electrolysis of a warm barium hydroxide solution in a glass tube fitted with two nickel electrodes. Oxygen was collected from the anode and supplied to the discharge tube after passing through concentrated sulphuricd acid. Hydrogen collected from the cathode was passed through electrically heated copper spiral and then through a series of U tubes containing solid caustic potash and phosphorous pentoxide. The main discharge current is placed at a suitable value depending upon the pressure. To find whether any Hall voltage developes at the two brass electrodes when the magnetic field is applied, it was observed that no current flows through the microammeter when no voltage was applied to the two plates even for magnetic fields as large as 200 G, hance for these low density plasma the Hall effect can be neglected. The experiment consists in measuring the variation of current flowing in the auxiliary circuit when some voltage is applied to the brass electrodes with the variation of the magnetic field, keeping the pressure constant. The experiment has been repeated for different pressures for each gas. It has to be particularly noted that the application of the magnetic field changes the value of the main discharge current but the discharge voltage has been so adjusted as to keep the discharge current constant for a particular value of pressure.

3. Result and discussion

The variation of the current in the direction perpendicular to the main discharge current with magnetic field has been plotted in case of oxygen, hydrogen and air in Figures (2–4). It is in general observed that in case of hydrogen, the current

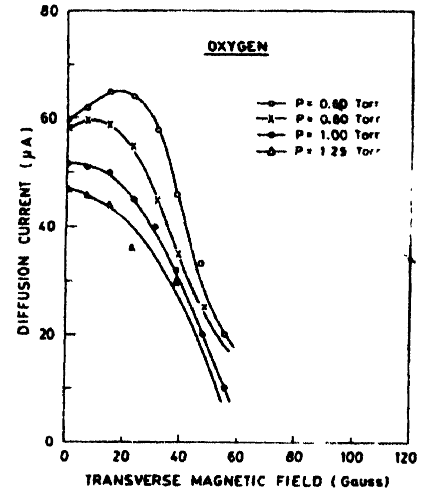

Figure 2. Variation of current with the magnetic field for oxygen

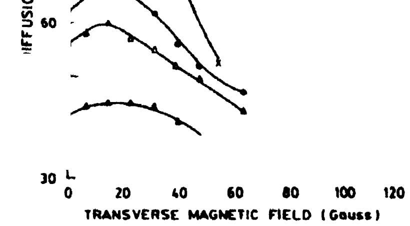

Figure 3. Variation of current with the magnetic field for hydrogen

gradually rises with the increase of the magnetic field, and after reaching the maximum value, gradually decreases. The value of the magnetic field at which the current becomes a maximum is almost the same at all pressures studied. In case of oxygen also, similar behaviour is observed for pressure smaller than 800 mT but beyond this pressure the current decreases gradually at the initial stage and then rapidly with the increase of the magnetic field without showing any maximum. Similar is the behaviour observed in the case of

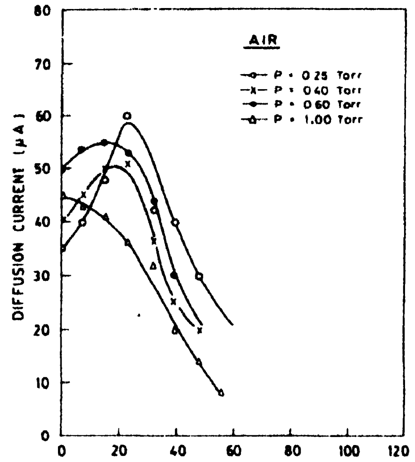

Figure 4. Variation of current with the magnetic field for air.

air for pressure smaller then 1000 mT and above this pressure the current gradually diminishes with the increase of the magnetic field. The variation of current in a transverse magnetic field in a glow discharge has been satisfactorily explained previously by Sen and Gupta [10] where it has been assumed after Beckman [11] that the effect of transverse magnetic field is to increase the axial voltage and reduce the radial electron density distribution. It has been shown by Beckman [11] that the axial electric field increases from E to E

$$[\alpha + \beta^2/\alpha^2]^{1/2}$$

where $\quad \alpha = 1 - h^2 + h^4 \exp h^2 \int_h^{\alpha} \frac{\exp(-h)}{h} dh$

and $\quad \beta = \frac{h}{2}\sqrt{\pi}\left[1 - 2h^2 + 4h^3 \exp h^2 \int_h^{\alpha} \exp(-h^2)dh\right]$

and $\quad h = \frac{eH\lambda_e}{m\omega}$

where H is the magnetic field, λ_e the electronic mean free path and ω the most probable electronic speed given by

$$\omega^2 = \frac{2KT_e}{m}$$

It has also been shown by Sen and Gupta [10] that the axial electric field increases in the presence of the transverse magnetic field, and, if E_H is the axial electric field in the presence of the magnetic field and E the electric field in its absence,

then $\quad E_H = E(1 + C_1 H^2/P^2)^{1/2}, \qquad (3)$

where C_1 is the constant for a particular gas and is given by

$$C_1 = \left(\frac{e}{m}\cdot\frac{L}{v_r}\right.$$

where L is the electronic mean free path at a pressure of 1 torr,

and v_r is the random velocity of the electron.

Further, it has been deduced by Sen and Gupta [10] that the field at which the current will be maximum will be given by

$$H_{max} = 12.41 * 10^{-2} \quad T_e \,)^{1/2} \qquad (4)$$

where k is the fraction of energy lost by collision and T_e is the electron temperature. Taking the value of T_e and k for different gases for the corresponding (E/P) values [13], H_{max} calculated for the three gases investigated, is shown in Table 1

Table 1. Values of H_{max}

Gas	H_{max} calculated from eq (4)	H_{max} expt value
Hydrogen	27.29	24
Oxygen	21.26	20
Air	20.36	19

Further, the experimental observation that the current decreases gradually without showing any maximum for higher values of pressure indicates that for small values of H/P, the above deduction cannot be regarded as valid. Based on the two assumptions that the effect of the magnetic field is to decrease the radial electron density and increase of the axial electric field it is noted that the current density

$$I = \frac{ne^2\lambda_e E}{mv_r}, \qquad (5)$$

where n is the electron density, λ_e the mean free path of the electron and v_r is the random velocity. It has been shown that in the presence of magnetic field, the axial electric field is changed to

$$E_H = E[1 + \omega_H^2/v_c^2]^{\frac{1}{2}},$$

where ω_H is the electron cyclotron frequency and v_c the collision frequency of the electron. Hence if n_H is the electron density in the presence of magnetic field, then the discharge current is

$$n_H e^2 \lambda_e \quad E[1 + \omega_H^2/v_c^2]^{\frac{1}{2}}, \qquad (6)$$

then from eqs. (5) and (6),

$$n_H/n : \frac{I_H}{I}\frac{1}{\sqrt{1 + \omega_H^2/v_c^2}}. \qquad (7)$$

In the region of pressure 0.1 to 10 torr where the diffusion theory holds, it has been shown by Schottky [12] that the loss of electrons and ions is entirely due to diffusion and

where J_0 is the Bessel function of order 0, n_0 is the axial density of electrons and v_i the production rate of ionization and D_a is the coefficient of ambipolar diffusion. In the balance of production rate v_i and duffusion D_a it has been shown by Brown [14] that

$$\frac{v_i}{D_a} \quad \frac{}{\Lambda^2},$$

where $\frac{1}{\Lambda} = \left[\left(\frac{\pi}{h}\right)^2 + \left(\frac{2.405}{R}\right)^2\right]^{1/2}$

where Λ is the diffusion length in the absence of magnetic field. Hence,

$$n = n_0 J_0(r/\Lambda) \tag{8}$$

and if Λ_{H} is the diffusion length in the pressence of magnetic field,

$$n_H = n_0 J_0(r/\Lambda_H). \tag{9}$$

Then from the eqs. (7–9), we have

$$\frac{I_H}{I\sqrt{1+\omega_H^2/\nu_c^2}} = \frac{J_0(r/\Lambda_H)}{J_0(r/\Lambda)}. \tag{10}$$

The values of ν_c has been determined experimentally from radio frequency conductivity measurements in the three gases studied here [15] and consequently, Λ_H/Λ can be obtained from various values of the magnetic field in these gases. The value of r is taken to that corresponding to a unit cross-sectional area around the axis of discharge tube *i.e.*

$\pi r^2 = 1$

or $r = 0.564$ cm.

The variation of Λ_H^2/Λ^2 with H^2/P^2 are plotted in Figures (5–7). It is observed that for low values of (H/P) and

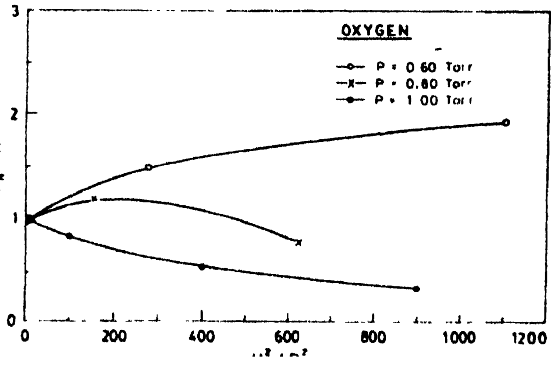

Figure 5. Variation of Λ_H^2/Λ^2 with H^2/P^2 for oxygen.

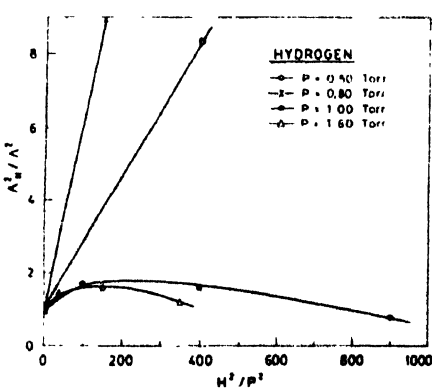

Figure 6. Variation of Λ_H^2/Λ^2 with H^2/P^2 for hydrogen

also for values of pressure less than 1 torr the relation between the two can be expressed by the formula

$$\Lambda_H^2/\Lambda^2 = \left[1 + C_1 H^2/P^2\right], \tag{11}$$

where C_1 is the constant. Assuming the diffusion in a direction transverse to the magnetic field represented by

$$D_H = \frac{D}{1+\omega_H^2/\nu_c^2},$$

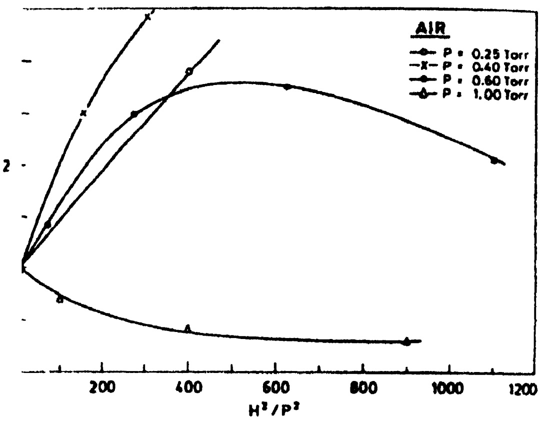

Figure 7. Variation of Λ_H^2/Λ^2 with H^2/P^2 for air.

an identical expression to eq. [11] has been deduced by Brown [14]

with $C_1 = \left(\frac{e}{m}\cdot\frac{L}{V_r}\right)^2$.

The results further, indicate that in case of gases studied here the linearity relation between Λ_H^2/Λ^2 and H^2/P^2 is valid only for small values of perferbly below 20 G/torr *i.e.*, for low values of maonetic field and high pressure and above these values the diffusion becomes anomalous. However, the entire mechanism responsible for anomalous diffusion is an open question. Nevertheless, it may be pointed out that in an weakly ionised gas a helical instability might have developed at relativey higher magnetic field causing enhanced radial losses and hence anomalous diffusion.

4. Conclusion

The values calculated theoretically for the magnitude of magnetic field at which the diffusion current becomes a maximum agree quite well with the experimental values for lower pressure. It can then be seen that the normal diffusion loss involving the inverse dependence upon the square of the magnetic field is valid for small values of H/P and above this the diffusion loss cannot be represented by a simple expression (1). No case of anomalous diffusion as noted by Lehnert [8] has been observed perhaps because of the low discharge current used in the present investigation.

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